

RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. THIRD SEMESTER EXAMINATION, MARCH 2021

SECOND YEAR [ARREAR]

MATHEMATICS FOR INDUSTRIAL CHEMISTRY [General]

Date : 30/03/2021

Time : 11 am – 1 pm

Paper : III

Full Marks : 50

Instructions to the Candidates

- Write your **College Roll No, Year, Subject & Paper Number** on the top of the **Answer Script**.
- Write your **Name, College Roll No, Year, Subject & Paper Number** on the **text body of your e-mail**.
- Read the instructions given at the beginning of each paper/group/unit carefully.
- Only handwritten (by blue/black pen) answer-scripts will be permitted.
- Try to answer all the questions of a single group/unit at the same place.
- All the pages of your answer script must be numbered serially by hand.
- In the last page of your answer-script, please mention the total number of pages written so that we can verify it with that of the scanned copy of the script sent by you.
- For an easy scanning of the answer script and also for getting better image, students are advised to write the answers on single side and they must give a minimum 1 inch margin at the left side of each paper.
- After the completion of the exam, scan the entire answer script by using Clear Scan: Indy Mobile App OR any other Scanner device and make a **single PDF file (Named as your College Roll No)** and send it to 

Group – A

Answer any three questions from Question nos. 1 to 5 :

[3×5]

1. Show that the area of the triangle formed by the straight lines $ax^2 + 2hxy + by^2 = 0$ and $lx + my = 1$ is

$$\frac{\sqrt{h^2 - ab}}{am^2 - 2hlm + bl^2}$$

2. Reduce the equation $x^2 - 3xy + y^2 + 10x - 10y + 20 = 0$, to canonical form. What is the nature of the curve?

[4+1]

3. If the straight line $r \cos(\theta - \alpha) = p$ touches the parabola $\frac{l}{r} = 1 + \cos \theta$, show that $p = \frac{l}{2} \sec \alpha$.

4. Find the shortest distance between the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-3}{3} = \frac{y-3}{4} = \frac{z-4}{5}.$$

5. Find the equation of the plane which passes through the point (2,1,4) and perpendicular to each of the planes

$$9x - 7y + 6z + 48 = 0 \text{ and } x + y - z = 0.$$

Group – B

Answer any four questions from Question nos. 6 to 11 :

[4×5]

6. a) Find the order and degree of $\left(\frac{d^2y}{dx^2}\right)^2 + \frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$. [2]

b) Eliminate the arbitrary constants A and B from the relation $y = \frac{A}{x} + B$ and obtain a differential equation of second order [3]

7. Solve : $\frac{dy}{dx} + \frac{y}{x} \log y = \frac{y}{x^2} (\log y)^2$

8. Find the general and singular solutions of the differential equation

$$y = px + ap(1-p), \text{ where } p = \frac{dy}{dx}.$$

9. Solve : $\frac{dy}{dx} = \frac{x+y+1}{2x+2y+1}$

10. Solve by method of variation of parameters

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x}.$$

11. Show that the orthogonal trajectories of $\frac{x^2}{a^2} + \frac{y^2}{a^2 + \lambda} = 1$, λ being arbitrary, is $x^2 + y^2 + c = 2a^2 \log x$.

Group – C

Answer any three questions from Question nos. 12 to 16 :

[3×5]

12. a) Examine if the set S is a subspace of $\mathbb{R}^3$ or not, where

$$S = \{(x, y, z) \in \mathbb{R}^3 : x = z = 0\}$$
 [3]

b) Determine k so that the set S is linearly dependent in $\mathbb{R}^3$, where $S = \{(1, 2, 1), (k, 3, 1), (2, k, 0)\}$. [2]

13. Show that $\{(1, 2, 3), (2, 3, 1), (3, 1, 2)\}$ forms a basis in $\mathbb{R}^3$.

14. Find a basis for the vector space $\mathbb{R}^3$, that contains the vectors $\{(1, 0, 1), (1, 1, 1)\}$.

15. Consider the map $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a linear transformation defined by $T(x_1, x_2, x_3) = (2a_2 + a_3, -a_1 + 4a_3, 5a_2)$. Find $[T]_\beta$, where $\beta = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$.

16. Consider the map $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(x_1, x_2, x_3) = (x_1 + 2x_2 + x_3, x_1 + x_2, 0)$

a) Show that T is a linear transformation.

b) Is T is one-one? Justify. [3+2]

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